COMPACTION OF ALIGNED FIBRE ASSEMBLIES USING PARTICULATE METHODS

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SUMMARY: A particulate approach is used for modelling the compaction behaviour of assemblies of aligned fibres. Fibre segments are represented as beams of reducing span subjected to 3-point bending as introduced by Gutowski. A more realistic relation between the fibre volume fraction and the displacement of the point of application of the contact force is introduced. This enables modelling of compaction up to the theoretical maximum fibre volume fraction corresponding to triangular alignment without the introduction of fitted parameters of the fibre volume fraction. The model is validated for simple assemblies and results for a larger array of fibres are presented. The particulate approach predicts more progressive stiffening with lower fibre volume fractions reached at practically meaningful compaction pressures.

KEYWORDS: compaction, yarns, fibres, particulate methods

INTRODUCTION

Compaction of reinforcements plays a central role in composites manufacturing. Models of compaction were developed [1] as parts of larger models of composites manufacturing processes [2]. Among those, Gutowski's compaction model [3] for aligned fibres is often used as it features straightforward analytical elements expressed in a closed-form equation that is easily implemented. The model is reviewed as a starting point for the work presented in this paper.

Gutowski's model has two main features. Firstly, fibre segments are represented as beams in 3-point bending; beam length reduces as compaction proceeds hence both the number of fibre-to-fibre contacts and the stiffness increase. Secondly, simple geometric relations are derived where the 1D displacement of the point of application of the force on the beam is transformed into a change of fibre volume fraction in the 2D section normal to the yarn length. The first feature was substantiated by Gutowski using imaging. The second feature brings limitations to the model. The fibre volume fraction v_f is approximated as the ratio of the volume of the fibre segment to that of a rectangular prism encompassing it. The height, length and in some cases the width of the

prism decrease as compaction proceeds, with the yarn represented as a series of prisms stacked regularly along their height, length and width. This regular stacking of fibre segments enables a closed-form equation but it is not representative of the different directions along which parallel fibres may interact in reality. It also imposes an unrealistic theoretical maximum on v_f equal to $\pi/4$ as shown below. Gutowski lifted this restriction by introducing fitted parameters. The resulting model describes the basic physical phenomenon that occurs during compaction and it is suitable to fit experimental data but the fitting parameters make it less analytical and more empirical. In this paper, particulate methods are used along with the first feature of Gutowski's model in developing a 2D model of compaction that better represents reality and uses less fitted parameters. The primary contribution is superior modelling of fibre positions and interactions. The approach is validated by comparing with Gutowski's model for simple assemblies of fibres, and results are presented for a more realistic assembly of fibres. Practical use of the results is discussed.

DEVELOPMENT

Gutowski's Compaction Model

Gutowski's model assumes that instead of being perfectly straight individual fibres in yarns undulate regularly in the vertical plane. A fibre segment submitted to 3-point bending as a result of contact with other fibres is illustrated in Fig. 1 with diameter d, length L, modulus E, second moment of inertia in bending I and contact force F. The relation between the vertical deflection δ and the contact force F at the centre of the segment is:

$$\delta = \frac{FL^3}{192E\mathrm{I}} \tag{1}$$

The length of the segment L and the deflection δ are defined as:

$$L = \beta\left(l - l_{\min}\right) \tag{2}$$

$$\delta = \left(l_o - l\right) \tag{3}$$

where l, l_{min} and l_o are the height, minimal height and initial height of the prism encompassing the fibre, and β is a parameter that is either evaluated by microscopy or fitted to experimental results. It is assumed that l_{min} is equal to d.

Two versions of the model's second feature were proposed, leading to two different versions of the model. In the first version the width of the prism w is assumed to stay equal to its height at all times during compaction ($w = l$). In the second version w is assumed to stay constant and equal to d ($w = d$). The following relations between the v_f, the initial fibre volume fraction v_o and the maximum fibre volume fraction v_a result for the first ($w = l$) and second ($w = d$) versions of the model respectively:

$$v_f = \frac{\pi d^2}{4l^2} \qquad\qquad v_o = \frac{\pi d^2}{4l_o{}^2} \qquad\qquad v_a = \frac{\pi}{4} \qquad (4)$$

$$v_f = \frac{\pi d}{4l} \qquad\qquad v_o = \frac{\pi d}{4l_o} \qquad\qquad v_a = \frac{\pi}{4} \qquad (5)$$

The relations between the pressure and the fibre volume fractions for the first ($w = l$) and second ($w = d$) versions of Gutowski's model are respectively:

$$P = \frac{F}{Lw} = \frac{3E\pi}{\beta^4} \cdot \frac{\sqrt{(v_f - v_o)} - 1}{\left[\sqrt{(v_a - v_f)} - 1\right]^4} \qquad\qquad P = \frac{F}{Lw} = \frac{3E\pi}{\beta^4} \cdot \frac{v_a \cdot \left(1/v_o - 1/v_f\right)}{\left[(v_a/v_f) - 1\right]^4} \qquad (6)$$

The maximum v_f allowed by both versions of the model is $\pi/4$ unless v_a is used as a fitted parameter. Effectively both v_a and v_o are used as fitted parameters along with β. The two versions of the model give comparable results as the force rises at v_f values that are close to v_a where differences in l at the same v_f between the two versions become negligible.
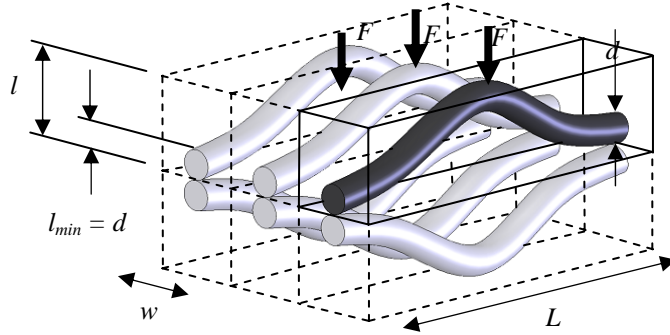

Fig. 1 Fibre segments and encompassing prisms in Gutowski's model.

Statistical Particulate Approach

Limitations related to the second feature of Gutowski's model are removed using a statistical particulate approach [4]. In the current approach a rectangular domain D is populated with n fibres positioned randomly. Fibres are tentatively displaced simultaneously from their positions within a circle of radius r, with $r < d/2$. The sum of force amplitudes E between pairs of fibres separated by a distance s with $l_{min} < s < l_o$ is calculated:

$$F_{i,j} = \frac{192EI}{\beta^3} \cdot \frac{\left(l_o - l_{i,j}\right)}{\left(l_{i,j} - l_{min}\right)^3} \qquad (7)$$

$$E = \sum_{i=1}^{n} \sum_{j=1}^{n} F_{i,j} \qquad\qquad \text{with } i \neq j \qquad (8)$$

$$l_{i,j} = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2} \tag{9}$$

$$F_{i,j} = F_{i,j} \qquad \text{if} \qquad l_{i,j} < l_0 \tag{10}$$

$$F_{i,j} = 0 \qquad \text{if} \qquad l_{i,j} \geq l_0 \tag{11}$$

where l_o is the initial distance between fibre centres, l_{min}, is again equal to the fibre diameter d, and x, y are the coordinates of fibre centres. Tentative positions are maintained if E is reduced by the displacements and the process is repeated until E is miminised. Compaction pressure is obtained:

$$P = \frac{F}{d_w \cdot L} \tag{12}$$

where d_w is the known domain width and the value of l in L is approximated as the minimum value of l for all pairs of fibres ($i = 1$ to n, $j = 1$ to n, $i \neq j$). Compaction is simulated by progressively reducing the domain height d_h. Fibres located near the top domain boundary are displaced, forces are re-equilibrated and P is re-calculated at each step.

Values of v_f are simply obtained by dividing the total cross-section area of n fibres by the cross section area of the domain. As the particulate approach is based on forces between pairs of fibres one should note that the resulting relation between δ and v_f is different from that of Gutowski at small n because the calculation of v_f is different. Fig. 2 compares a single fibre in the second version of Gutowski's model with the simplest case that can be handled by the particulate approach, one force between two fibres. The same initial distance between fibres l_o and fibre diameter d lead to different values of v_o for small n. The way in which v_f is calculated in Gutowski's model is useful in that context because it allows the scaling of the P-v_f relation obtained for a single fibre to a whole assembly of parallel fibres; the relation stays the same for any number of fibres. In our approach the relation converges rapidly as n increases. This expected convergence is irrelevant in practice as the particulate approach is aimed at large assemblies of fibres. This point of detail is only mentioned to address questions that may arise when comparing results from the simplest case that the particulate model can handle with those obtained from Gutowski's model, as discussed below.
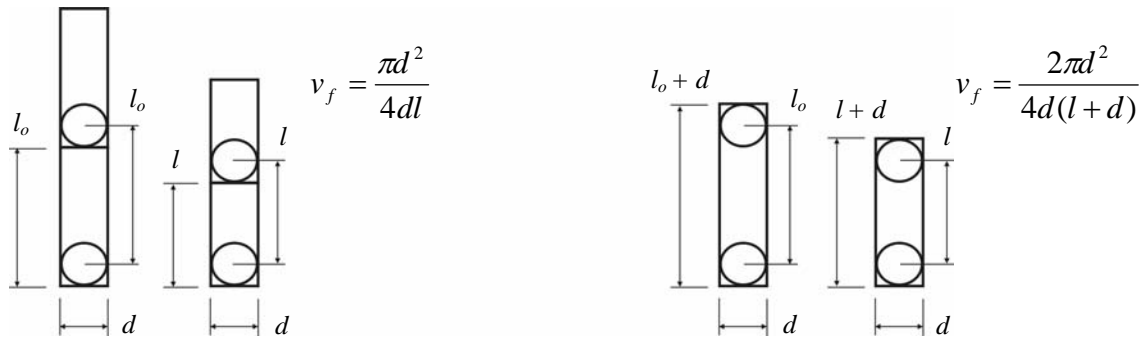

Fig. 2 Comparing v_f for 2nd version of Gutowski's model with particulate approach at $n = 2$.

VALIDATION AND COMPARISONS

Case 1: $n = 2$

A first validation was conducted for the case of 2 fibres shown in Fig. 2. Fig. 3 shows P-v_f curves for the particulate approach and the second version of Gutowski's model with $l_o = 50 \times 10^{-6}$ m, $l_{min} = d = 10 \times 10^{-6}$ m, $\beta = 280$ [3], $d_w = 10 \times 10^{-6}$ m and $d_h = 60 \times 10^{-6}$ m, up to P of approximately 6×10^6 Pa. For any given l, values of δ, F and P are the same in both cases whilst v_f is larger for the particulate approach. As expected, for $n = 2$ the particulate approach shows lower pressure P for the same v_f. Fig. 3 also shows a curve of Gutowski's model with v_f altered to account for the difference mentioned above. This curve superimposes with results from the particulate approach.

Case 2: Single Column, $w = d$, $n = 10$

A second validation was conducted for 10 fibres positioned in a single vertical column. Fig. 4 shows P-v_f for the particulate approach and the second version of Gutowski's model without alteration to v_f values, with $l_o = 50 \times 10^{-6}$ m, $l_{min} = d = 10 \times 10^{-6}$ m, $\beta = 280$, $d_w = 10 \times 10^{-6}$ m and $d_h = 51 \times 10^{-5}$ m, up to P of approximately 7×10^5 Pa. At any given pressure P, v_f is only marginally larger for the particulate method. This shows that the P-v_f curve converges rapidly with n and that the two models predict virtually similar behaviour for this simple case.

Case 3: Double Column, $w = 2d$, $n = 20$

A third validation was conducted to illustrate a specific aspect of the particulate approach. 20 fibres were positioned in two vertical columns with $d_w = 20 \times 10^{-6}$ m and $d_h = 51 \times 10^{-5}$ m. The domain is similar to twice Case 2 side by side; other parameters were unchanged. Fibres were seeded so that minimization would naturally lead to the staggered configuration shown in Fig. 5 which is more stable than aligned prisms in Gutowski's model. Fig. 5a shows P-v_f for the particulate approach and Gutowski's model up to $P = 3 \times 10^7$ Pa. As expected the behaviour at higher P is similar to that observed for Case 2 and to unaltered results of Gutowski's model in Case 1. Fig. 5b shows the same curve up to $P = 1 \times 10^5$ Pa and v_f of 0.6. At this stage of the compaction staggered fibres are closer to each other along the vertical though they are not perfectly aligned along the horizontal as in Gutowski's model. Therefore, P is larger in this more realistic configuration and increases more progressively at levels relevant to VARTM and RTM.

Case 4: Double Column, $w = 2d \cdot \cos(30°)$, $n = 20$

A last validation was conducted to illustrate an important aspect of the particulate approach. A total of 20 fibres were positioned in two vertical columns with $d_w = 18.66 \times 10^{-6}$ m and $d_h = 51 \times 10^{-5}$ m. Other parameters were unchanged. The domain has the same initial height than that of Case 3, but its width corresponds to two rows of fibres aligned in a triangular configuration (Fig. 6). These small domains lead to simple compaction behaviour: fibres are forced into square and triangular arrays in Cases 3 and 4. Case 4 illustrates that the particulate method allows larger v_f values. Fig. 6a shows P-v_f for the particulate approach and Gutowski's model up to $P = 1.5 \times 10^9$ Pa. Below $v_f \cong 0.75$ Case 4 gives larger P than Gutowski's model as discussed in Case 3. Beyond $v_f \cong 0.75$, P from Gutowski's model quickly rises towards infinite at $v_f = 0.785$. The curve

generated using the particulate approach raises more progressively towards infinite at $v_f = 0.906$. The latter behaviour is more realistic, with P limited to 1.0×10^5 Pa for VARTM and 1.0×10^6 Pa for RTM.
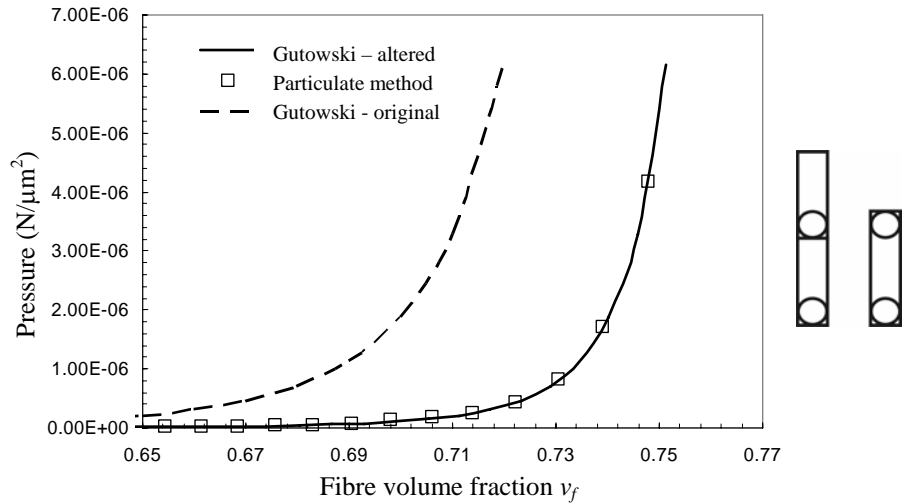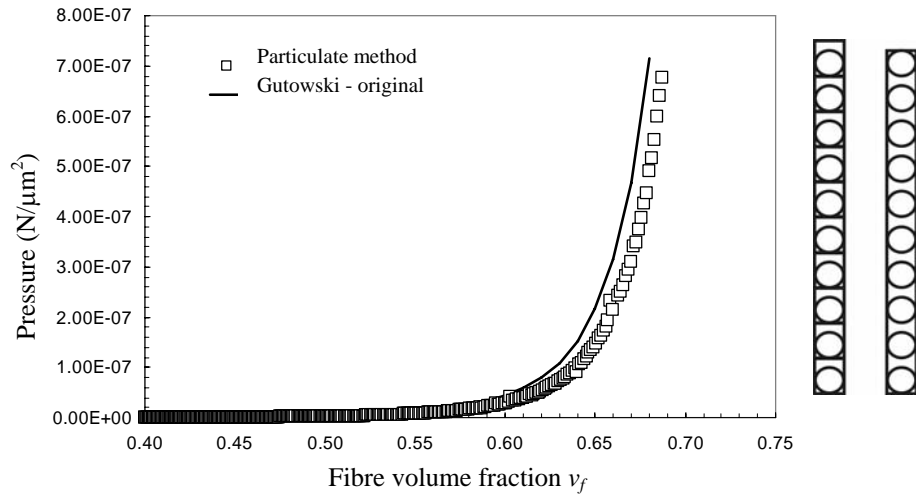

Fig. 3 Compaction curves, Case 1.

Fig. 4 Compaction curves, Case 2.

RESULTS

An assembly of 25 particles was compacted using the particulate approach with parameters as above. The compaction curve appears in Fig. 7 whilst Fig. 8 shows initial fibre positions, positions after equilibrium and positions during compaction. At compaction pressures P representative of liquid moulding processes the stiffening of the fibre assembly is progressive, with v_f values below 0.7. It should be noted that in this case the fibre volume fraction is underestimated by approximately 10% as a result of the combined natural alignment of fibres and artificial nature of the straight boundaries. Such restrictions will fall when the particulate approach are combined to more extensive geometric descriptions of reinforcement unit cells.
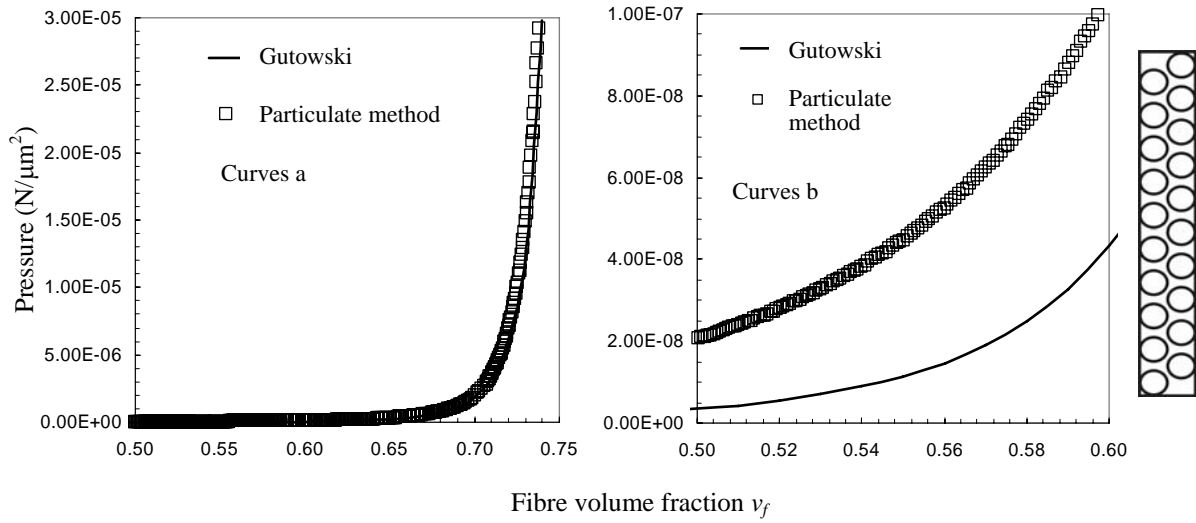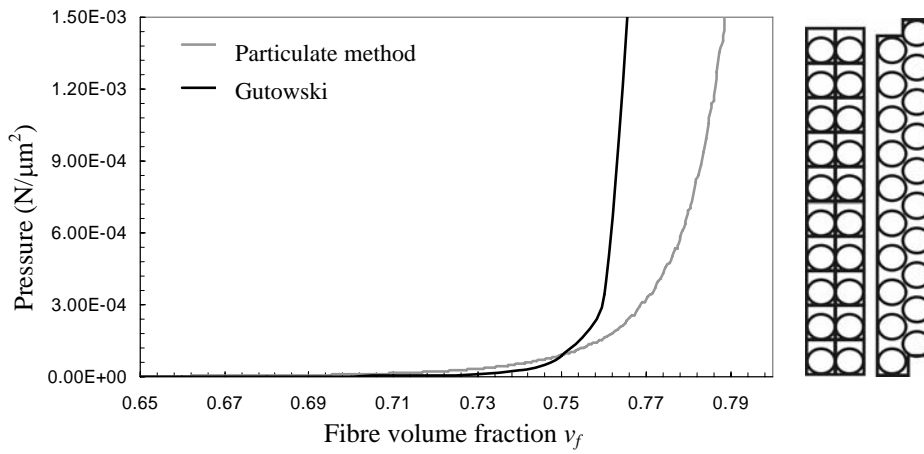
Fig. 5 Compaction curves, Case 3.

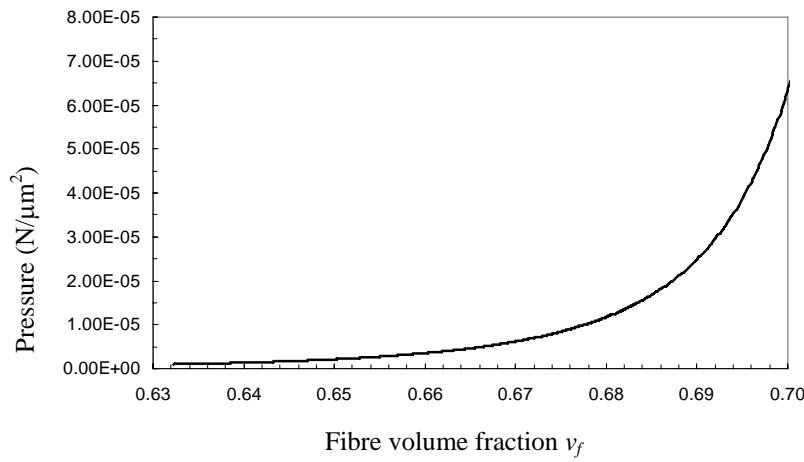
Fig. 6 Compaction curves, Case 4.

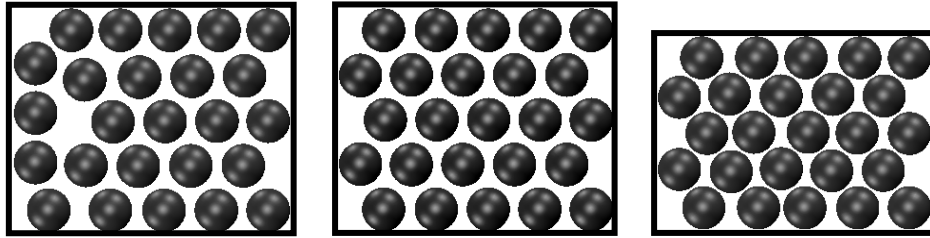Fig. 7 Compaction curve for 25 fibres.

Fig. 8 25 fibre positions: initial , 1st equilibrium, during compaction (left to right).

CONCLUSION

In this paper a statistical particulate approach was applied to the compaction of assemblies of parallel fibres as found in yarns. The particulate approach was validated using simple cases and demonstrated for a larger array of fibres. Interaction between 2 fibres is described in a similar way as in Gutowski's well-known model, though other relations could also be used. The main contribution of this work is a better representation of the relative positions of fibres and their interactions. At compaction pressures representative of those encountered in VARTM or RTM the approach predicts more progressive stiffening with v_f values lower than those predicted by Gutowski's model. The basic P-v_f relation derived from the model can be fitted to a simple equation and used in larger models of composites manufacturing processes.

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